MONYETLA NOTES SEQUENCES AND SERIES 7/02/2025

June 2017:

QUESTION 3

Given the quadratic sequence: 0; 17; 32; ...

3.1 Determine an expression for the general term, T_n, of the quadratic sequence. (4)

3.2 Which terms in the quadratic sequence have a value of 56? (3)

3.3 Hence, or otherwise, calculate the value of $\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$. (4)

[11]

June 2017:

Q3.1

First differences: 17; 15

Second difference: -2

$$T_n = an^2 + bn + c$$

$$a = \frac{\text{second difference}}{2} = -1$$

$$3a + b = 17$$

$$3(-1) + b = 17$$

$$b = 20$$

$$a + b + c = 0$$

$$-1 + 20 + c = 0$$

$$c = -19$$

$$T_n = -n^2 + 20n - 19$$

Q3.2

$$56 = -n^2 + 20n - 19$$

$$n^2 - 20n + 75 = 0$$

$$(n-15)(n-5) = 0$$

$$n = 5 \text{ or } n = 15$$

Q3.3

$$\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$$

$$= T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} - T_{11} - T_{12} - T_{13} - T_{14} - T$$

$$= (T_5 - T_{15}) + (T_6 - T_{14}) + \ldots + (T_9 - T_{11}) + T_{10}$$

$$= T_{10}$$

because by symmetry $T_5 = T_{15}$; $T_6 = T_{14}$ ...

$$T_{10} = -(10)^2 + 20(10) - 19$$

$$= 81$$

Nov 2019

QUESTION 3

3.1 Without using a calculator, determine the value of: $\sum_{y=3}^{10} \frac{1}{y-2} - \sum_{y=3}^{10} \frac{1}{y-1}$ (3)

Nov 2019

Q3.1

$$\sum_{y=3}^{10} \frac{1}{y-2} - \sum_{y=3}^{10} \frac{1}{y-1}$$

$$= \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{8}\right) - \left(\frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{8} + \frac{1}{9}\right)$$

$$= 1 - \frac{1}{9}$$

$$= \frac{8}{9}$$

EXAMPLE:

1.1 Write in $\sum$ notation ie. $\sum$ general term

3 ; 7 ; 11 ; to 20 terms

$$a=3 \quad d=4$$
$$T_n = a+(n-1)d$$
$$=3+(n-1)4$$
$$=3+4n-4$$
$$=4n-1$$
$$\sum_{k=1}^{20} 4k-1$$

1.2 Write in $\sum$ notation

$3\ ;\ 6\ ;12$ to 20 terms

$$a=3; \quad r=2$$
$$T_n = ar^{n-1}$$
$$=3(2)^{n-1}$$
$$\sum_{k=1}^{20} 3(2)^{k-1}$$

1.3 Hence write the following in $\sum$ notation

$$1\ ;\frac{7}{6};\frac{11}{12}$$

$$\frac{3}{3};\frac{7}{6};\frac{11}{12}$$

$$\sum_{k=1}^{20} \frac{4k-1}{3(2)^{k-1}}$$

March 2017:

3.2 Determine the value(s) of x in the interval $x \in [0^\circ ; 90^\circ]$ for which the sequence $-1\ ;\ 2\sin 3x\ ;\ 5\ ;\ .....$ will be arithmetic. (4)

Q3.2

$$-1\ ;\ 2\sin 3x\ ;\ 5;\ \ldots$$

$$2\sin 3x+1=5-2\sin 3x$$

$$4\sin 3x=4$$

$$\sin 3x=1$$

$$3x=90^\circ$$

$$x=30^\circ$$

May-June 2019

2.2 Given a geometric sequence: $36\ ;\ -18\ ;\ 9\ ;\ldots$

2.2.1 Determine the value of r, the common ratio. (1)

2.2.2 Calculate n if $T_n=\dfrac{9}{4\,096}$ (3)

2.2.3 Calculate S_∞ (2)

2.2.4 Calculate the value of $\dfrac{T_1+T_3+T_5+T_7+\ldots+T_{499}}{T_2+T_4+T_6+T_8+\ldots+T_{500}}$ (4)

[17]

MAY /JUNE 2019

Q2.2.1 $r=\dfrac{-18}{36}=-\dfrac{1}{2}$ **Q2.2.2** $T_n=36\left(-\dfrac{1}{2}\right)^{n-1}$ $\dfrac{9}{4096}=36\left(-\dfrac{1}{2}\right)^{n-1}$ $\dfrac{1}{16384}=\left(-\dfrac{1}{2}\right)^{n-1}$ $\left(-\dfrac{1}{2}\right)^{14}=\left(-\dfrac{1}{2}\right)^{n-1}$ $14=n-1$ $n=15$	**Q2.2.3** $S_\infty=\dfrac{a}{1-r}$ $=\dfrac{36}{1-\left(-\dfrac{1}{2}\right)}$ $=24$ **Q2.2.4** $\dfrac{T_1+T_3+T_5+T_7+\ldots+T_{499}}{T_2+T_4+T_6+T_8+\ldots+T_{500}}$ $=\dfrac{a+ar^2+ar^4+\ldots+ar^{498}}{ar+ar^3+ar^5+\ldots+ar^{499}}$ $=\dfrac{a+ar^2+ar^4+\ldots+ar^{498}}{r(a+ar^2+ar^4+\ldots+ar^{498})}$ $=\dfrac{1}{r}$